VŠB – Technical University of Ostrava
Faculty of Mechanical Engineering
Department of Applied Mechanics

Návrh experimentu pro identifikaci frekvenčně závislých mechanických vlastností pryže

Design of an Experiment to Identify the Frequency-Dependent Mechanical Properties of Rubbers

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VŠB - Technical University of Ostrava
Faculty of Mechanical Engineering
Department of Applied Mechanics

Diploma Thesis Assignment

Student: **Bc. Jorge Alberto Rodríguez García**

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The thesis language: English

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2. Goal and requirements of the proposed variant of the measurement method.
3. CAD/CAE design of the measurement specimen considering the available measurement equipment.
4. Verification of usability frequency band and a possibly variation of the specimen dimension for covering whole required frequency range.

References:

[1] COOK, Robert D. *Finite element modeling for stress analysis*. Vyd. 1. Hong Kong: John Wiley, 1995, 320 s. ISBN 04-711-0774-3.
[2] BELYTSCHKO, Ted, W. K. LIU a B. MORAN. *Nonlinear finite elements for continua and structures*. New York: Wiley, c2000. ISBN 0-471-98773-5.
[3] *ANSYS® Academic Teaching Advanced, Release 18.2, help system, ANSYS, Inc.*

Extent and terms of a thesis are specified in directions for its elaboration that are opened to the public on the web sites of the faculty.

Supervisor: **Ing. Michal Weisz, Ph.D.**

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VYSOKÁ ŠKOLA BÁŇSKÁ TECHNICKÁ UNIVERZITA Fakulta strojní -4- OSTRAVA

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Head of Department

prof. Ing. Ivo Hlavatý, Ph.D.
Dean

Student's affidavit

I declare that I have prepared the whole diploma thesis including appendices independently under the leadership of the diploma thesis supervisor, and I stated all the documents and literature used.

In the thesis, I used internal information about the technical parameters and basics of damping properties identification from the company BONATRANS GROUP a.s., the company agrees to their disclosure.

In Ostrava on May 20, 2019.

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ANNOTATION OF DIPLOMA THESIS

RODRÍGUEZ GARCÍA, J. *Design of an Experiment to Identify the Frequency-Dependent Mechanical Properties of Rubbers: Diploma Thesis*. Ostrava: VŠB – Technical University of Ostrava, Faculty of Mechanical Engineering, Department of Applied Mechanics, 2019, 48 p. Thesis head: Weisz, M.

This diploma thesis work is a part of wheel damper development and is based mainly on the proposal of a piece of metal for measurement of dynamic damping properties. The objective is to create a modal analysis based on the idea of the piece that is currently used by the company, and in addition to them, to propose some other form or model that is more viable and meaningful to be implemented in wheel damper system development. The FEM modal analysis of proposed dimension and mass variants is performed and based on the results the most promising candidates are chosen for dynamic parameters verification, which is done by FEM frequency response analysis. The obtained results are used to assess the dynamic properties of chosen candidates to the dynamic ability of the shaker LDS V650, which is expected for dynamic testing.

ANNOTACE DIPLOMOVÉ PRÁCE

RODRÍGUEZ GARCÍA, J. *Návrh experimentu pro identifikaci frekvenčně závislých mechanických vlastností pryže: diplomová práce*. Ostrava: VŠB – Technická univerzita Ostrava, Fakulta strojní, Katedra aplikované mechaniky, 2019, 48 s. Vedoucí práce: Weisz, M.

Tato diplomová práce je součástí vývoje tlumiče kol a je založena na návrhu kové části vzorku pro měření dynamických tlumících vlastností. Cílem je vytvořit modální analýzu vycházející ze stávajícího provedení tohoto dílu, a dále pokračovat návrhem jiné formy nebo modelu, který je životaschopnější a smysluplnější pro další rozvoj procesu návrhu tlumičů kol. Je provedena MKP modální analýza navrhovaných rozměrových a hmotnostních variant a na základě výsledků jsou vybrány nejslibnější návrhy pro ověření dynamických parametrů, což je provedeno analýzou frekvenční odezvy pomocí MKP simulací. Získané výsledky jsou použity k posouzení dynamických vlastností vybraných návrhů vzhledem k dynamickým možnostem budiče LDS V650, který bude použit pro zamýšlené dynamické testování

CONTENT

LIST OF USED DESIGNATIONS

Symbol	Unit	Description
{a(t)}	[m/s^2]	Unknown vector of nodal acceleration
[C]	[-]	Known damping matrix
d	[m]	Nodal displacement
E	[Pa]	Young modulus
E_k	[J]	Kinetic energy
E_p	[J]	Potential energy
f	Hz	Frequency
{F(t)}	[s]	Known vector of nodal loads as a function of time
F (ω)	[Hz]	Function of frequency
[F]	[-]	Known vector of nodal knowns
G	[Pa]	Shear modulus
[K]	[N/m]	Known stiffness matrix
m	[kg]	mass
[M]	[kg]	Known mass matrix
{u(t)}	[m]	Unknown vector of nodal displacements
{v(t)}	[m/s]	Unknown vector of nodal velocity
η	[-]	Loss factor
$[\varphi]_i$	[-]	Eigen vector
$\omega^2{}_i$	[-]	Eigen value

LIST OF USED ABBREVIATIONS

ASTM	American Society for Testing and Materials
DOF	Degree of freedom
ISO	International Organization for standardization
FE	Finite element
FEA	Finite element analysis
FEN	Finite element methods
FRF	Frequency response function
SDOF	Single degree of freedom

LIST OF FIGURES

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1 INTRODUCTION

Wheels are an essential component of any type of rolling stock. GHH-BONATRANS is engaged in production of railway wheelsets, wheels, axles and hoops for all types of rail vehicles, is one of the world's leading manufacturers of wheels of all types of rolling stock with a history dating back to 1808. The wheels run on high-speed trains and conventional trains, subway cars, trams, locomotives and freight cars.

Both the demands of customers, and the standards for the railway wheels, are increasingly demanding. In order to ensure good and efficient operation of rail transport, each wheel must not only be skillfully developed, but before that it must be intelligently designed. And this is where we stand out, thanks to the experience of many years and our deep knowledge supported by the leading research and development centers.

The specimen design is developed under the requirement of the GHH-BONATRANS. This diploma thesis work is a part of wheel damper development and is based mainly on the proposal of a piece of metal for measurement of dynamic damping properties. The metal piece function is to carry a layer of damping material, mostly rubber or composite, which damping properties will have determined by the mentioned measurement.

The objective is to create a modal analysis based on the idea of the piece that the company has, and in addition to them, to propose some other form or model that is more viable and meaningful to be implemented in wheel damper system development. To create that model, I relied on a proposal from the company and from there, it has begun to develop the metal piece.

In this master thesis will be consider the methodology according to American standard ASTM E756-05: 2010 Standard Test Method for Measuring Vibration-Damping Properties of Materials which has been so far used by cooperating supplier. The damper research and development process pointed out some weak spots of the methodology in connection to the plate damper specific attributes. Although the thesis refers to the standard methodology, the work is done by numerical finite element analysis and a kind of design of experiment study is a part of the thesis.

2 Determination of Dynamic Damping Properties

The various parts of ISO 6721 specify methods for the determination of the dynamic mechanical properties of rigid plastics within the region of linear viscoelastic behaviour. This part of ISO 6721 is an introductory section which includes the definitions and all aspects that are common to the individual test methods described in the subsequent parts.

This method is based on the measurement of vibration and damping properties of materials, namely loss factor η, Young's modulus of elasticity in tension E or shear modulus G. It is used when testing materials that are used in vibration structures, building acoustics and audible noise control. Materials to be tested include metals, enamels, ceramics, rubber, plastics, reinforced epoxy matrices and wood, from which it is possible to create test pieces in the shape of a bar.

Different deformation modes may produce results that are not directly comparable. For example, tensile vibration results in a stress which is uniform across the whole thickness of the specimen, whereas flexural measurements are influenced preferentially by the properties of the surface regions of the specimen. Values derived from flexural-test data will be comparable to those derived from tensile-test data only at strain levels where the stress-strain relationship is linear and for specimens which have a homogeneous structure.

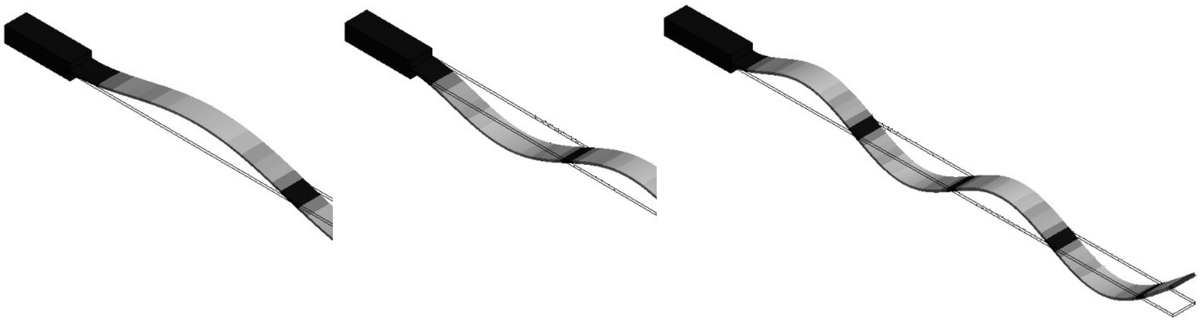


Figure 1 – Flexural deformation usually expected within the measurement

According to the standard, only the bending is specified, whilst the wheel will really have more complex vibrations shape, not only in-plane bending but also in combination of bending and share component. The following pictures in Fig. 2 show the real vibration behaviour of the plate damper mounted to the wheel under simulation of typical wheel operational conditions.

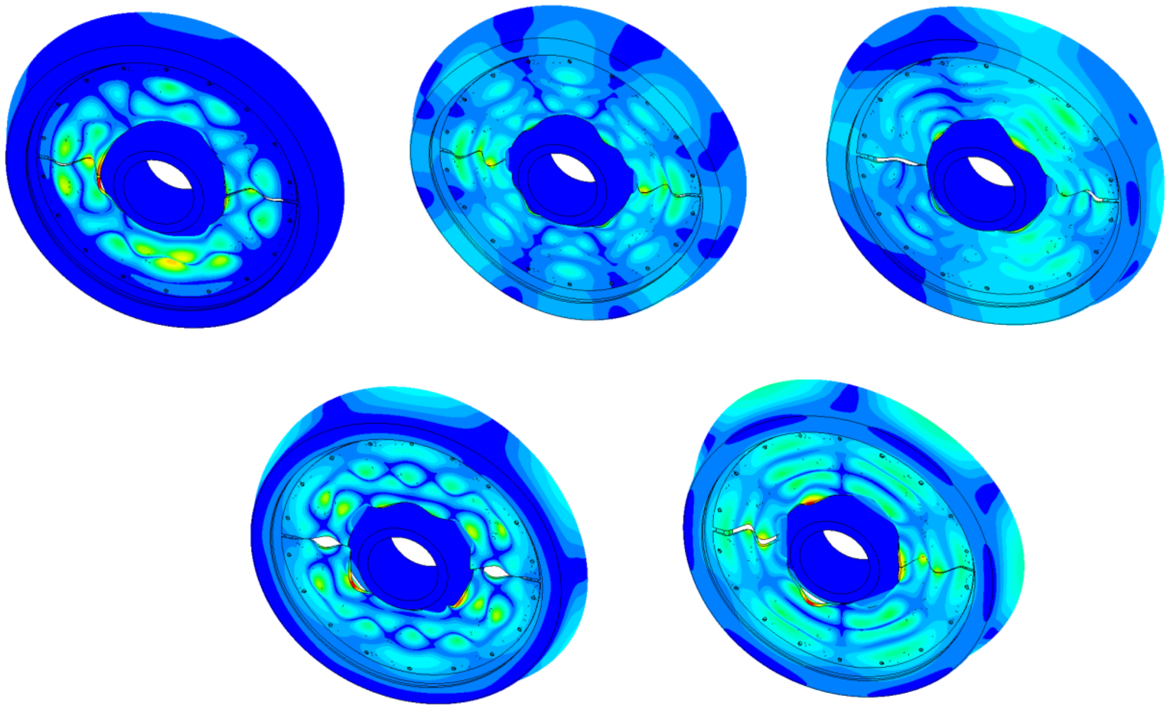


Figure 2 – Variations of vibrations of wheel plate damper

The differences in dynamic deformations between the measurement specimen and real piece of damper can be noticed by comparing the pictures in Fig. 1 and 2. As the whole plate damper is to heavy to be vibrated at the shaker the particular goal of my work is to propose rather light specimen plate which allows to achieve dynamic deformation shapes similar to those observed at real damper vibration conditions.

The final dynamic testing with the specimen proposed by this work shall be carried out at the VSB-TUO laboratory and the shaker type LDS V650 (Fig. 4) will be involved. To perform this testing experiment, two pieces of specimen will be mounted on the shaker, one opposite the other, at the place where the supports are located. The next picture Fig. 3 indicates a simple sketch of the shaker and how the specimens arrangement could look like.

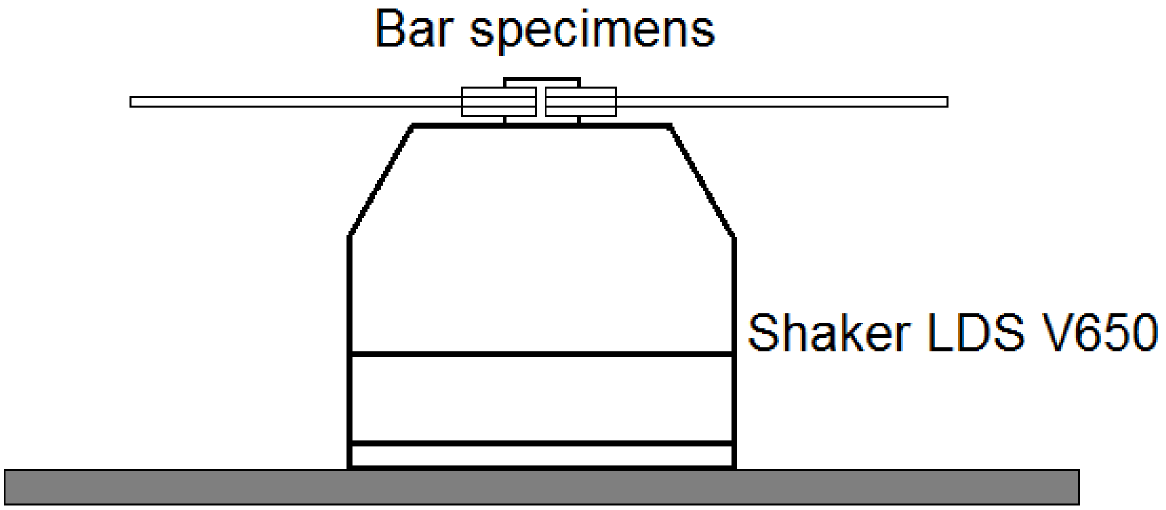


Figure 3 – Scheme of shaker test and specimen arrangement

Figure 4 – Shaker LDS V650

3 FEA THEORETICAL BASIC PRINCIPLES

Finite element analysis, commonly referred to as FEA, is a tool of design analysis. Therefore, it is appropriate to start our discussion with a definition of what design analysis is and how it relates to FEA.

Design analysis is a process of investigating certain properties of parts, assemblies, or structures. Design analysis can be conducted on real objects or on models that represent certain aspects of a real object. If models are used instead of real objects, the analysis can be conducted earlier in the design process, before the final product or even the prototypes are built. Those models can be physical models (e.g., scaled-down models, mockups, photoelastic models) or mathematical models where a certain behaviour of a part or structure is captured and described by a mathematical apparatus. The design analysis conducted with the use of mathematical models can be further broken down, based on what methods are used to obtain the solution. Simple mathematical models can be solved analytically, but more complex models require the use of numerical methods.

Finite element analysis is one of those numerical methods used to solve complex mathematical models. It has numerous uses in science and engineering but in this work we will focus on its applications to dynamic structural analysis as used in the field of mechanical engineering. We will alternate between two terms that became synonymous in engineering practice: (1) finite element analysis, and (2) finite element method (commonly referred to as FEM).

3.1 Modelling

FE modelling is the simulation of physical behaviour by a numerical process based on pieceweise polynomial interpolation. In order to obtain a reliable FE solution, the analyst must first have a grasp of the problem area, be in this case modal analysis.

If FE analysis goes astray it is usually because the analyst´s understanding of physical behaviour, boundary conditions, limitations of theory, FE behaviour or options in the program is insufficient to prepare a satisfactory model. FE model is more than preparing a mesh and pre-processing

3.2 Structure and element behaviour

What type of element should be used? (beam, shell, solid, etc) Triangular or quadrilateral, with or without side nodes, mid-side nodes, how many nodes?, how should the mesh be graded, are there no linearities? Such questions are inevitably arisen. Answers may not come easily, especially for the initial FE model, but will not come at all without some understanding of how the structure is likely to behave and how elements are able to behave. In general, one remembers that the essense of the FE method is piecewise polynomial interpolation and tries to select elements of such a type and size that deformation of the structure over the region spanned by an element is closely approximated by deformation modes that the element can represent.

3.3 Element tests and element shapes:

How do elements of various shapes behave under various loads? A good way to find out is by computational testing, choosing problems for wich the solution is already known. By doing computational tests we may incidentally learn how to use the software more effectively, and also resolve uncertainties about input conventions, defaults, output capabilities, coordinate systems used for stress output, symbols and abbreviations, and explanations in the documentation. A single-element test is a FE analysis like any other, except that the model consists of a single element. By varying the aspect ratio L/h one can determine the sensitivity of an element to elongation. If the element stiffness matrix is numerically integrated, the effect of changing the quadrature rule can be studied (if the software permits a choice in the matter).

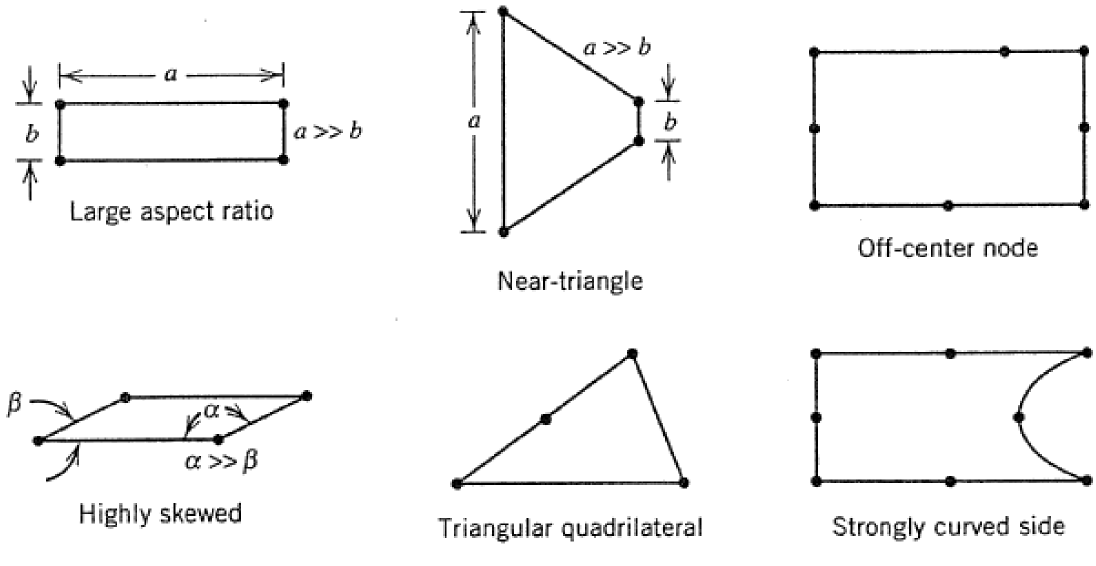


Figure 5 – Plane elements having shapes distortions that usually reduce accuracy

Element shapes that are compact and regular usually give greatest accuracy. Accordingly, the ideal triangle is equilateral, the ideal quadrilateral is square and so on.

3.4 Loads:

A concentrated load must be applied at a node. This is required by practice, not theory. With the possible exception of beam elements, standard software is not structured to accept non-nodal concentrated loads as an input data. In practice one merely arranges the mesh so that there is a node at each location where a concentrated load must be applied.

According to classical linear theories of beams, plates and solids, at a point loaded by concentrated normal force there is:

1) finite displacement and finite stress in beam
2) finite displacement and infinite stress in a plate
3) infinite displacement and infinite stress in a two or three dimensional solid.

3.5 Connections

Connections between parts can made by bolting, welding, gluing and so on. Realistic modelling of connections is usually difficult, with FE or any other analysis method, because of geometric complexity and the possibilities of slippage, gap closure, and partial loss of contacts. A fine-mesh FE model of a connection may capture its behaviour accurately, but such a model is not practical unless the connection itself is the object of study. More often connections are modelled only to the extent needed to represent their effect on the rest of the structure. If data about connection stiffness are known, perhaps from experiment, one might approximate a connection by using standard elements that have modified elastic properties.

3.6 Boundary conditions

Also called support conditions in structural mechanics. They are often misrepresented or not described properly as input data. Care is needed because changes in support conditions that appear minor can have a major effect on computed results.

Some support conditions are dictated by FE technology rather than by physical considerations. A restraint such a prescription of zero rotation must appear at a node rather than between nodes. The DOF (common abbreviation for degree of freedom) not active in de FE model must be suppressed, whether or not they are on the boundary of the FE model.

Boundary conditions are often misrepresented because of carelessness or because the physical situation does not present a clear choice.

3.7 Modal analysis

Modal analysis, also called "frequency analysis," finds natural frequencies and the modes of vibration that are associated with those frequencies. The body vibrates in the absence of damping and excitation force.

Note that although any real-life structure has an infinite number of degrees of freedom, it still has discrete modes of vibration. Each mode, with its frequency and the associated modal shape, corresponds to the situation where elastic stiffness forces cancel out with inertial forces. Therefore, the mode of vibration (or the eigenvector) may be understood as the shape that the body vibrating with a certain frequency must assume in order for inertia forces to cancel out elastic stiffness forces. The resultant stiffness of the structure vibrating in resonance is zero, and the only factor controlling the amplitude of vibrations is damping. [2]

We again recall the fundamental FEM equation that is applicable to static analysis:

$$[K]\cdot\{u\} = \{F\}. \tag{1}$$

To consider dynamic effects, the equation must be extended to account for inertial and damping effects and for the fact that load can be a function of time Eq. 1:

$$[M]\cdot\{a(t)\} + [C]\cdot\{v(t)\} + [K]\cdot\{u(t)\} = \{F(t)\}, \tag{2}$$

where:

[M] = Known mass matrix
[C] = Known damping matrix
[K] = Known stiffness matrix
{F(t)} = Known vector of nodal loads as a function of time
{a(t)} = Unknown vector of nodal acceleration
{v(t)} = Unknown vector of nodal velocity
{u(t)} = Unknown vector of nodal displacements

The modal analysis deals with free and undamped vibrations where $\{F(t)\} = 0$ (no excitation force) and $[C] = 0$ (no damping). Therefore, Eq. 2 can be rewritten as:

$$[M]\cdot\{a(t)\} + [K]\cdot\{u(t)\} = 0\ . \tag{3}$$

Non-zero solutions of Eq. 3 present an eigenvalue problem and provide with modal frequencies and associated modal shapes of vibration:

$$[K]\ [\varphi]_i = \omega^2{}_i[M]\ [\varphi]_i\ . \tag{4}$$

The last relationship Eq. 4 has n solutions, where $\omega^2{}_i$ is called the "eigenvalue," and the corresponding vector $[\varphi]_i$ is called the "eigenvector." The relation between the eigenvalue and the frequency expressed in Hertz [Hz] is:

$$f = \frac{\omega_i}{2\pi}\ . \tag{5}$$

3.8 Frequency response analysis

All types of analyses that have been discussed so far assume that load is not a function of time. Now this restriction will be lifted to discuss some common types of dynamic analysis, but first we must clarify an important terminology issue. Strictly speaking, the term "dynamic analysis" applies to the analysis of unconstrained bodies and mechanisms, whereas "dynamic analysis" within the scope of FEA deals with vibrations about the equilibrium. A more appropriate term to use would be "vibration analysis," but the term "dynamic analysis" is so well entrenched in the FEA literature that we will use it, understanding that dynamic analysis within FEA really means the analysis of the vibrations of structures. We will briefly discuss only two of the most common types of linear dynamic analysis: (1) time response analysis, and (2) frequency response analysis.[2]

Frequency response analysis assumes that the load is a function of frequency rather than being directly dependent on time, as was the case in dynamic time analysis:

$$[M]\cdot\{a(\omega)\} + [C]\cdot\{v(\omega)\} + [K]\cdot\{u(\omega)\} = \{F(\omega)\}, \tag{6}$$

Frequency response models structure response to force excitation or base excitation (excitation applied to support) that is a sine function of time. Frequency response analysis also uses the modal superposition method and requires that damping be defined, most often as a percentage of critical damping.

One typical application of frequency response analysis is a simulation of a shaker table test. For instance, some hammer is installed on a shaker table and is subjected to a frequency sweep from 0 to 300 Hz. Only modes with frequencies within this range have a chance to contribute to the dynamic response. However, taking into account the direction of the base excitation, only modes 1 and 4 actually contribute to the dynamic response. For some cases when during the frequency sweep the excitation frequency passes through the resonant frequency (calculate in the prerequisite modal analysis), the amplitude of vibrations peaks because it is controlled only by damping.

The following figure shows a general explanation about the kinetic and potential energy contained in each of the bars. When the maximum amplitude is shown in the sine wave, the kinetic energy will be zero, but the potential energy at that point will be the maximum that the body contains.

On the contrary, it happens when the wave's point of inference coincides with the *x* axis, the kinetic energy is maximum at that point and the potential energy is zero.

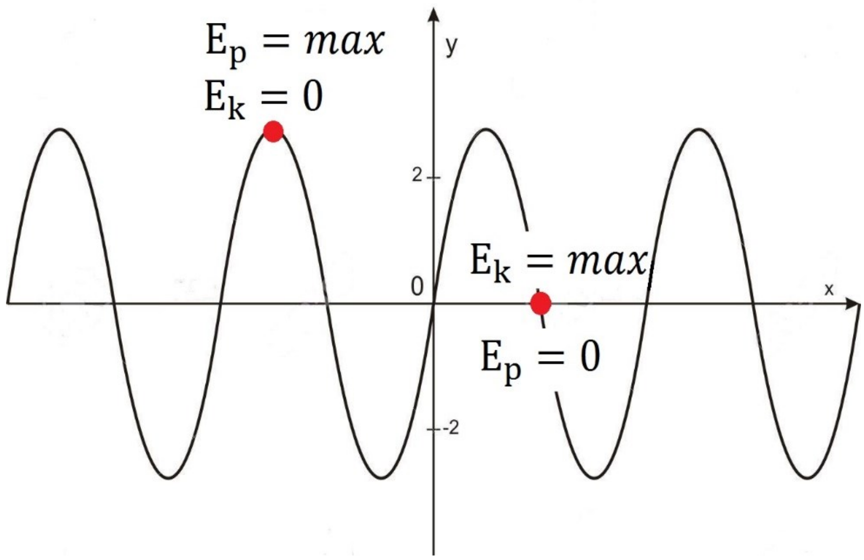


Figure 6 – Kinetic energy and potential in a sine wave

3.9 Mode superposition method

A review of dynamic analysis must be preceded by a short description of the modal superposition method, on which both time response and frequency response analyses are usually based. The modal superposition method represents the dynamic response of a vibrating structure by superposition of responses of several single-degree-of-freedom (SDOF) systems. The natural frequencies of those single-degree-of-freedom systems correspond to natural frequencies of the analysed structure. The number of single-degree-of-freedom systems contributing to dynamic response is equal to the number of modes calculated by the prerequisite modal analysis. How many modes should then be calculated to represent the dynamic response using the modal superposition method? The first few modes are most important, but the exact number of required modes is not known prior to the analysis. Ideally, one should demonstrate in a convergence process that increasing the number of modes past a certain number no longer significantly affects results.

The modal superposition method is not always a prerequisite for dynamic analysis. Other methods, such as the direct integration method, do not require modal analysis results. However, those methods have very limited application in the FEA performed in a design environment [2]

4 BASICS OF MEASUREMENT ARRANGEMENT

The following chapter describes the methodology for measuring the vibration and the small comparison of a material according to ASTM E756-05: 2010 - Standard Test Method for Measuring Vibration - Damping Properties of Materials.

For this case the specimen design is developed under the requirement of the GHH-BONATRANS. Under the proposal of a piece of metal on the wheel of the tram, which will help to reduce vibrations in this part and braking is done more effectively.

To create that model, I relied on a proposal from the company and from there, it has begun to develop the piece.

The Fig. 7 shows a bit how it looks the train wheel which includes the metal part that will be worked with.

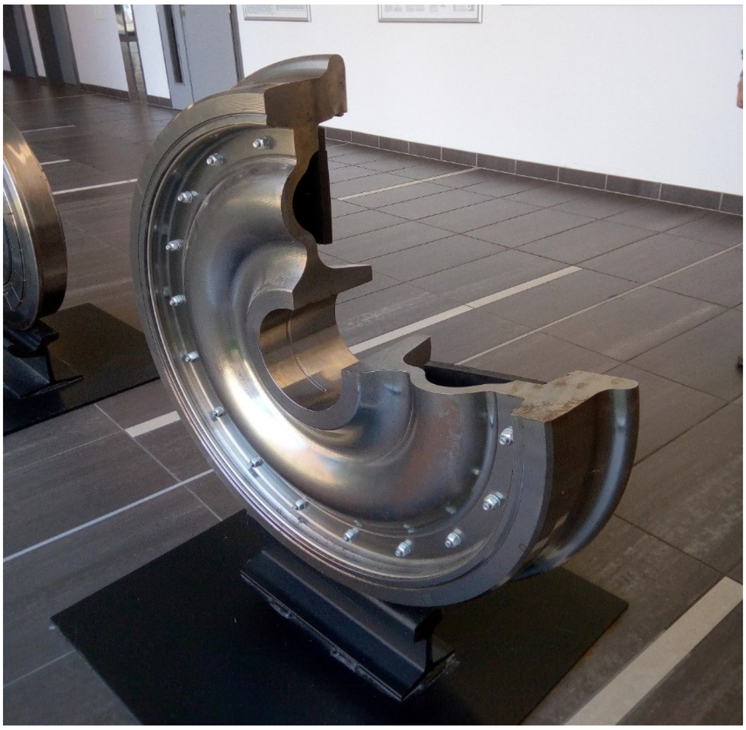


Figure 7 – BONATRANS Tram Wheel

The set of metal bars are placed along the circumference. The following Fig. 8 shows a sample that is part of the entire rubber disc cut of the wheel and can be seen inside of the wheel where the metal bars are housed.

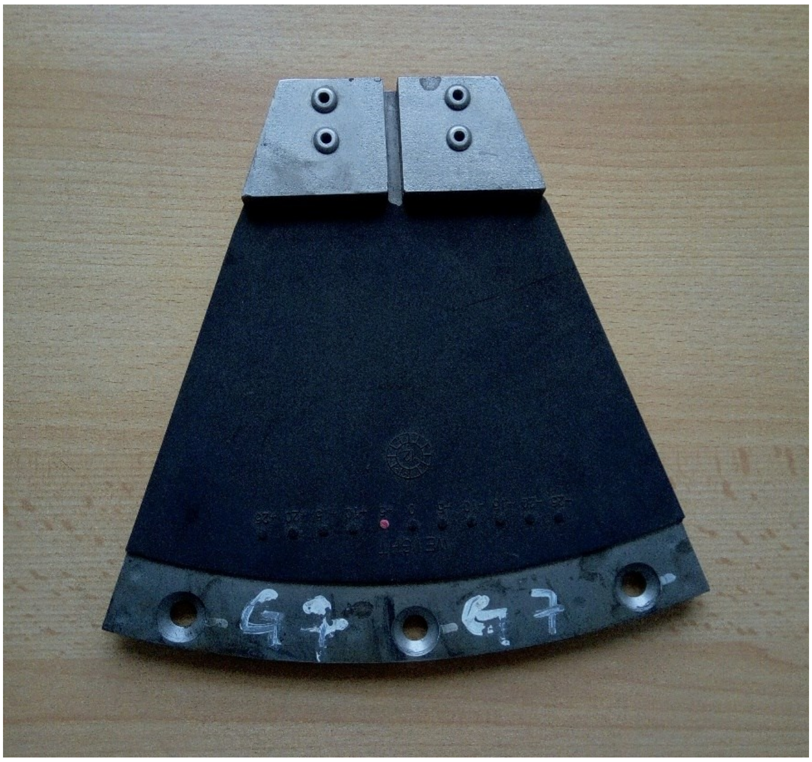
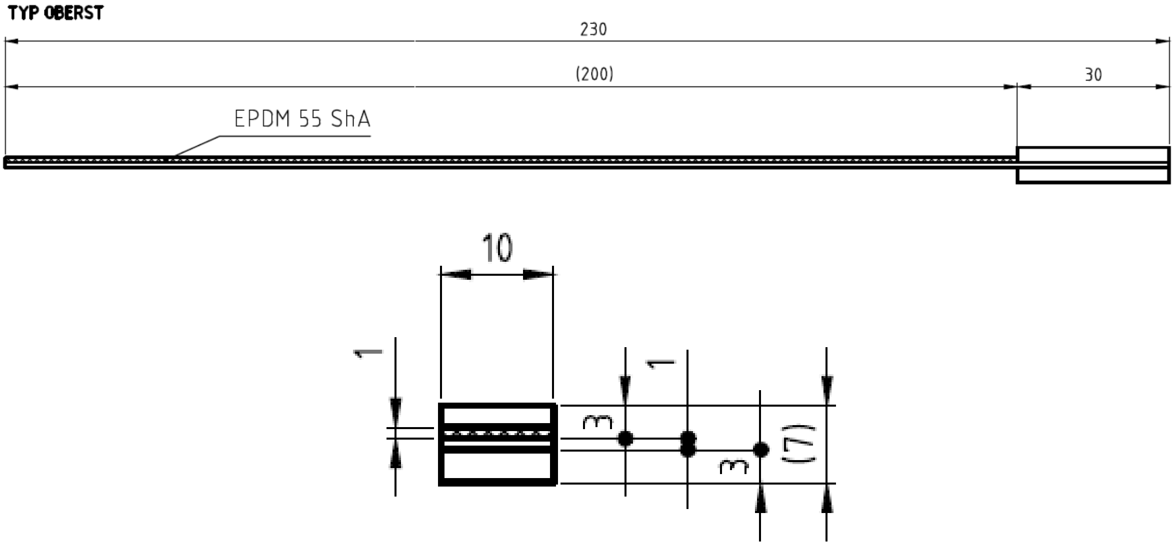


Figure 8 – Cut of the wheel

Figure 9 – Transversal cut of the rubber disc

4.1 Basic concepts and principles

It is a measure against structural vibrations by applying a layer of damping material to the surface of the structure, so that the energy is dispersed during the cyclic deformation of the damping material. This method uses fixed dimensions of recommended dimensions as test specimens. The configuration of the test beam sample is selected according to the type of damping material to be tested and the damping parameters to be determined. The drawings in Fig. 10 show two different test specimens used to investigate the material's tensile-pressure and shear properties for a wide range of module values E and G. The first one has the dimensions given by BONATRANS, and the second ones the proposed by me, just the double of the width.



Sample extended with double width

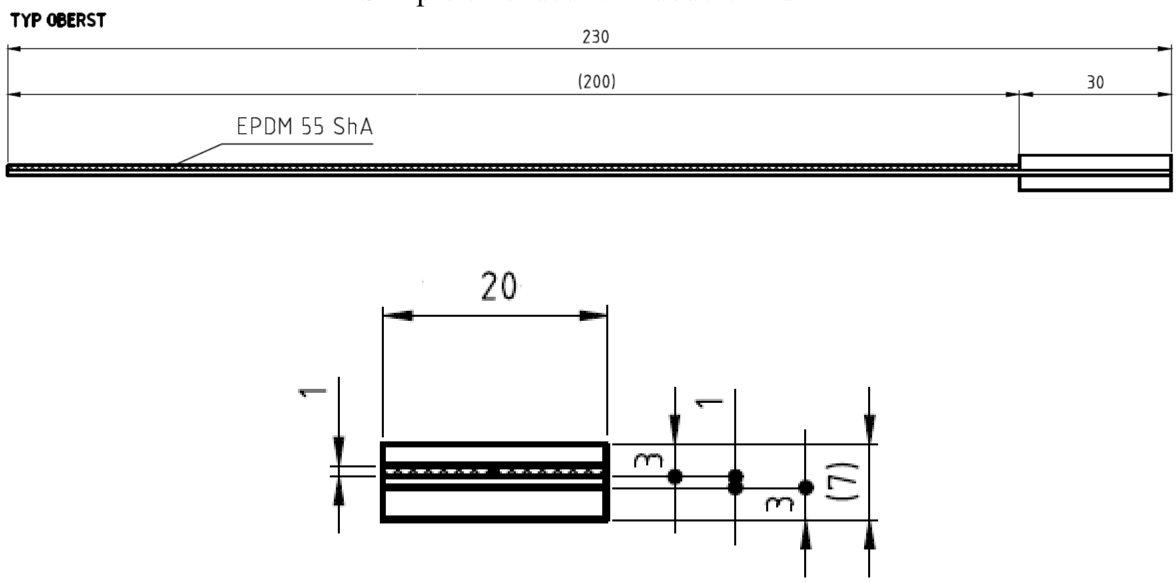


Figure 10 – Dimensions of the specimens

The measurement is carried out on a simple test beam made in ANSYS WORKBENCH.

To determine frequency parameters, proceed in two steps. First for the specimen proposed by the company which can be called a base bar, must be measured to determine its resonant frequencies. In the next step, same calculations are done but for the new specimen proposed by us, in order to see the behavior and after the result, give the comparison and ensure wich could be the most suitable for BONATRANS.

Both beams, under the configuration of being held from one side called "sandwich" sample is created by a homogeneous contact at the end of one extreme.

The width of the may vary for each particular case calculating the material properties plays no role, but the beam thickness is important as torsional vibrations may occur.

The loss factor and the modulus of elasticity of the damping materials are useful parameters in designing measures to reduce the structural vibrations and energy that propagate through these structures. The results and material properties can be used in mathematical models to design damping systems and predict their efficiency before they are manufactured for future usage by BONATRANS. This applies to both the analytical models of simple beams and plates, and to finite element models of complex structures.

The test materials are composed of one homogeneous layer, this method could give good results. In case of complicated designs, the individual layers must be tested separately if we want the mathematical model prediction to be as accurate as possible.

4.2 Assumptions

All measurements are made in a linear region of the material where the material behaves according to linear viscoelastic theory. If the linear area of the material is exceeded, the analysis data is unusable. The upper limit of the linear range occurs when the size of the system loss factor from two consecutive measurements differs more than the accuracy of the measuring system.

The analysis does not include the effect of moments of inertia and shear deformation and assumes that the area of the planar section remains planar.

The beam test technique is based on measuring the differences between the base and proposed bar. Both beams are tested without damping according to the requirement from BONATRANS.

The ASTM E756-05: 2010 methodology has been applied to determine the elastic modulus of elasticity of elastic E and the elastic modulus of shear G, which uses fixed beams to measure the vibration and damping properties of the material.

5 VARIATIONAL STUDY OF SPECIMEN BASE SHEET DESIGN

Three types of bars are used for this analysis. The first is called model A and corresponds to the base bar, whose dimensions are equal to the model provided by BONATRANS (see figure 4). The second type of bar will only have double the width, keeping the thickness and length, is called model B. The third proposal has a variation of a thickness of the supports and the bar but keeping the length of the previous two and the width of the second, the bar is called model C. Figures 5, 6 and 7, show the sketch model of each bar.

To better understand the determined dimensions of each bar model, a table 1 with the measurement specifications is attached.

	Bar		
	Model A	Model B	Model C
Lenght (mm)	230	230	230
Width (mm)	10	20	20
Thickness (mm)	1	1	3
	Suport "sandwich"		
Lenght (mm)	30	30	30
Width (mm)	10	20	20
Thickness (mm)	3	5	5

Table 1. Specific measures of the bars

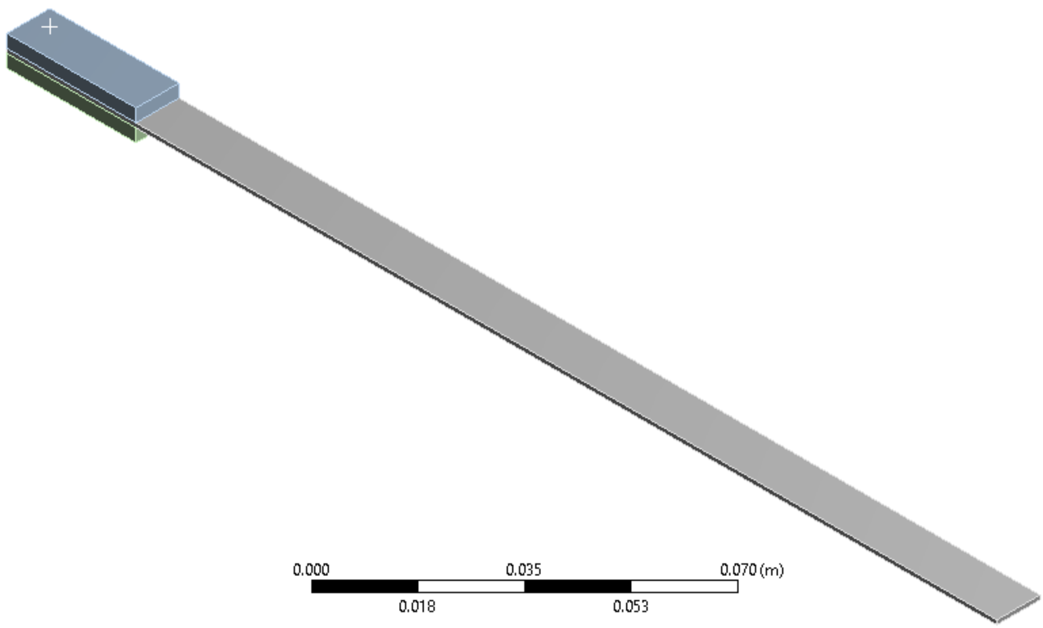


Figure 11 – Bar model A

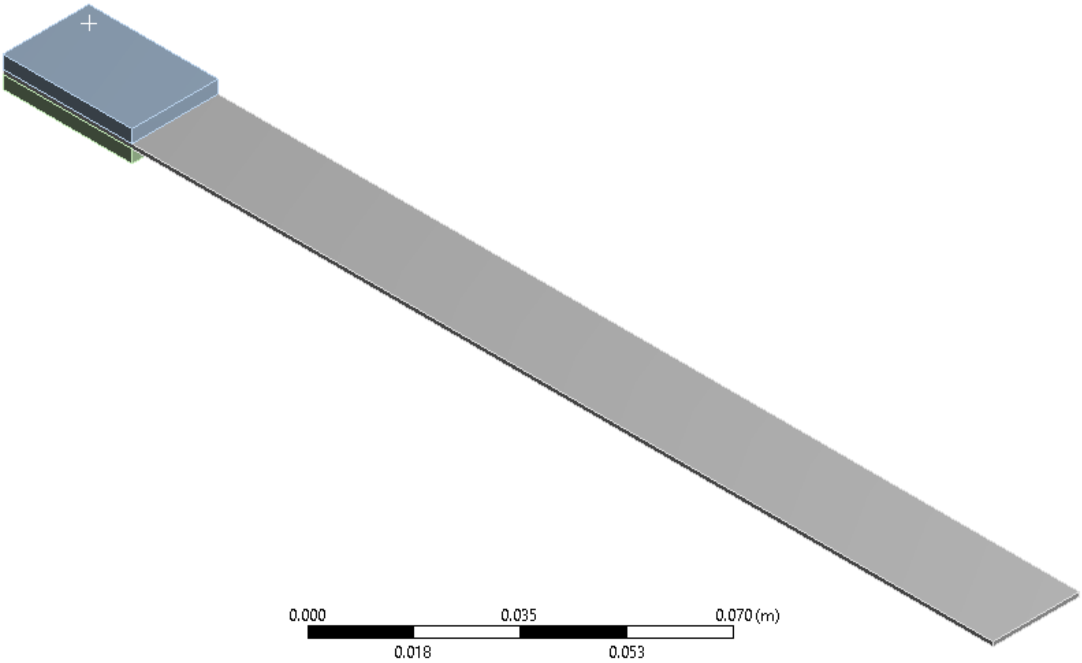


Figure 12 – Bar model B

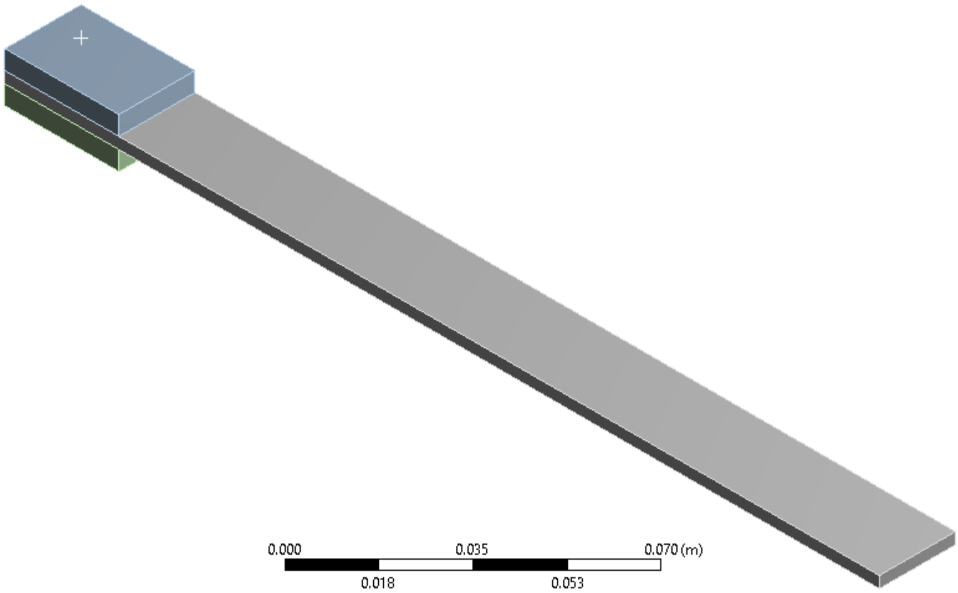


Figure 13 – Bar model C

To carry out a modal analysis, this is done using each of the models.

Apart of this, it has been proposed to add small masses distributed along each bar, for each of the mentioned models. For this, 4 types of mass distribution have been designed. The figure shows the types of mass distribution.

All models (A, B, C) will have the same mass distribution, the only variation is the particular dimension of each bar model. The base model A is used as an example to indicate the types of mass distribution of the base bar. See figure 8.

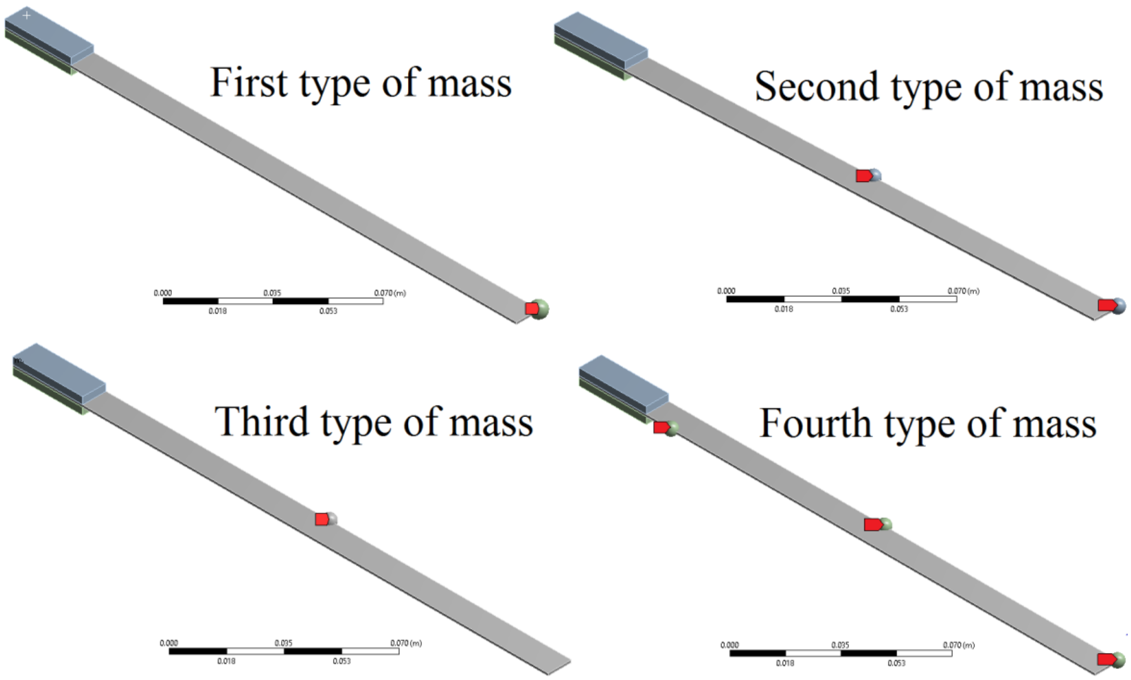
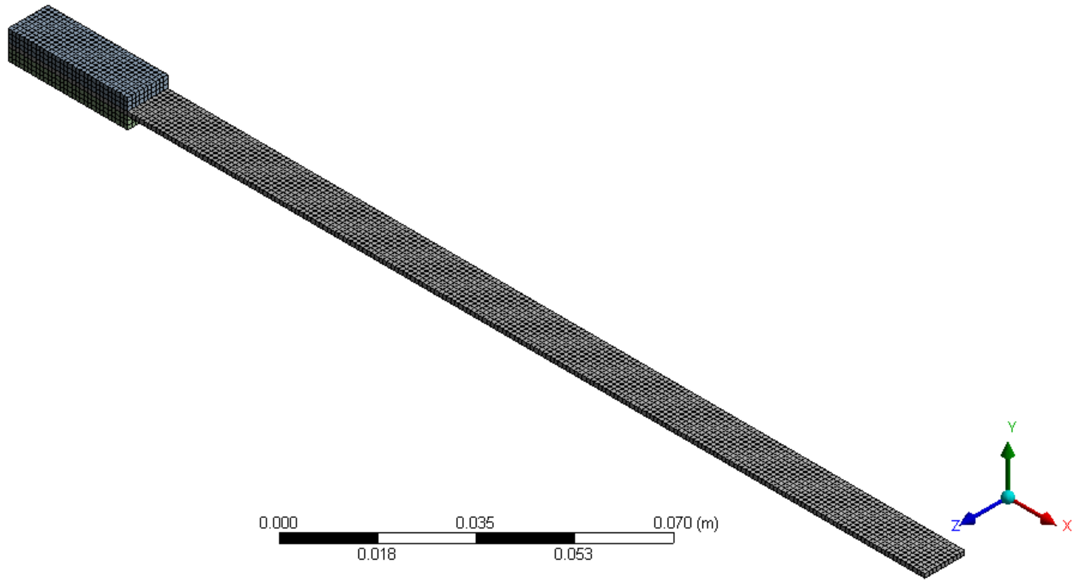


Figure 14 – Types of mass distribution

It is important to mention again that the objective of adding the masses is to find the highest dissipation of energy of each of the bars at the moment in which a vibration is simulated.

5.1 Boundary conditions for the analysis

For this 3D sketch is applied a mesh to the geometry for the modal analysis, the following figure 7, shows the meshing corresponding to the base and extended bar.



Mesh of the extended bar

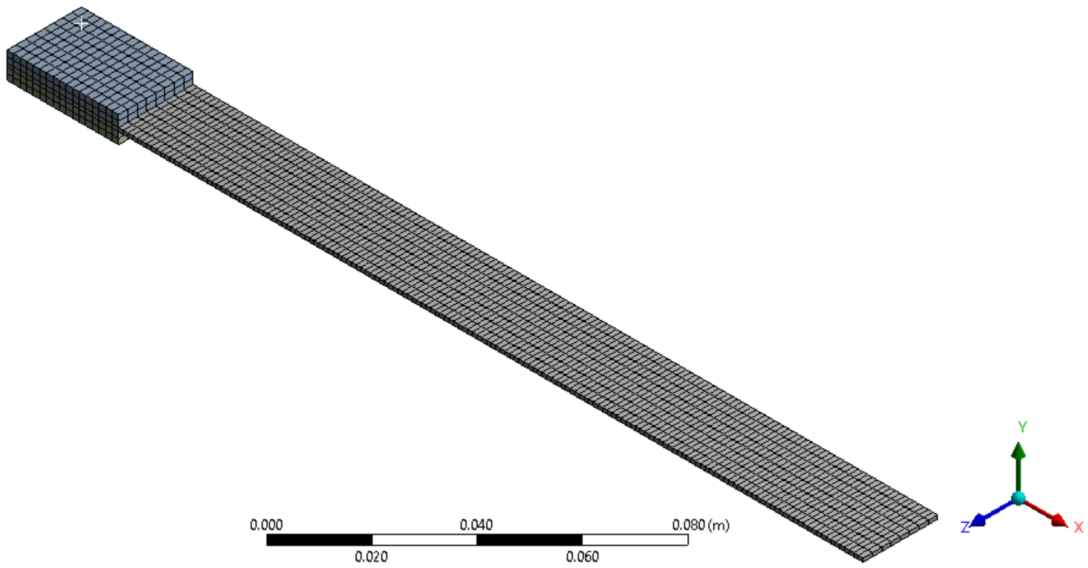


Figure 15 – Meshing of base and extended bar

As the specimen indicates, exist two holders that act as a support like a "sandwich" for both beams. So for this simulation is added at one of the extremes a fixed support in the top two faces of the "sandwich" which simulate the way it would be subject to some staples at the moment in which the vibration will go to be done in the shaker that is in the laboratory (LDS V650) for further tests. The figure 7 shows the fixed support for both bars.

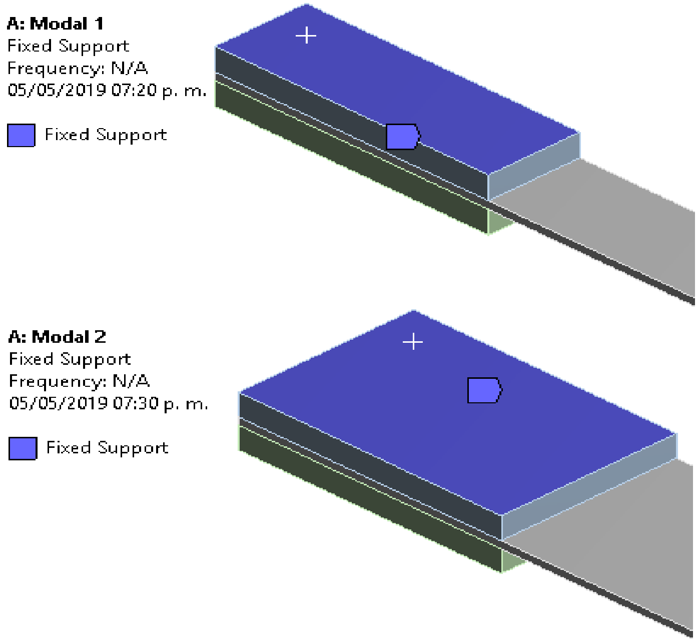


Figure 16 – Fixed support

The range for the evaluation is located between 0 and 6500 Hz during the modal analysis process. The damping parameter it is not considered for this calculations as a requirement of BONATRANS

5.2 Performance of the analysis

Once the design adjustments have been made, as well as the border conditions the results of all the models (A, B, C), combined with the four different types of distribution, gives a lot of data, which have been subjected to a comparison between modal forms. The set of results thrown by the tests are comparable by pairs of bars, making all the possible combinations with the aim of finding the one that is the most appropriate to the result that must be reached.

5.3 Table results comparison

Of all the results set, the one that meets most closely with the requirement was selected to be able to build the appropriate rubber for the damper. The best result obtained is that of pair model A and B type of mass 4.

Firstly, the results from the modal analysis of the base beam are in total of 24 mode shapes. The table 4 shows the frequencies, kinetic energy and modals mass for each mode shape. On the other hand you get the result of the ratio between effective mass and total mass from column 5 to 10, which includes the bendings in all axis, and their rotations See table 1.

Nr.	Freq [Hz]	Kin. energy [J]	Mod. mass[kg]	x_dir	y_dir	z_dir	rx_dir	ry_dir	rz_dir
1	5.221	31.3287	5.82E-02	0.000011	0.476891	0	0.002793	0	0.931509
2	34.015	1382.5114	6.05E-02	0.000008	0.152997	0.000004	0.007349	0.000005	0.046116
3	51.524	3064.9435	5.85E-02	0.000828	0.000001	0.479349	0.013123	0.930821	0
4	237.365	3998.1207	3.60E-03	0.000259	0.005584	0.000295	0.007899	0.000059	0.000667
5	328.666	88405.2316	4.15E-02	0.002414	0.023905	0.136853	0.11346	0.039875	0.001944
6	345.524	18726.7622	7.95E-03	0.000915	0.156999	0.01836	0.51122	0.006171	0.012619
7	568.017	31487.7302	4.94E-03	0.000967	0.085201	0.000701	0.195731	0.000075	0.005454
8	734.091	19453.5681	1.83E-03	0.003546	0.000809	0.000132	0.028887	0.000003	0.000026
9	885.176	47472.1108	3.07E-03	0.013026	0.001735	0.000246	0.004751	0.000089	0.000088
10	1419.215	101697.62	2.56E-03	0.031544	0.000474	0.002242	0.008961	0.00035	0.000011
11	1485.519	1384242.156	3.18E-02	0.368199	0.000019	0.001597	0.000192	0.000054	0.000009
12	1941.648	235643.6121	3.17E-03	0.202461	0.000038	0.026801	0.000384	0.004534	0
13	2119.367	147900.1424	1.67E-03	0.041567	0.000266	0.013471	0.000048	0.00184	0.00002
14	2235.712	303541.1776	3.08E-03	0.000385	0.000032	0.036144	0.002551	0.002803	0.000001
15	2565.035	989756.2294	7.62E-03	0.000189	0.000414	0.114334	0.001331	0.008473	0.000028
16	3134.661	376847.0712	1.94E-03	0.001266	0	0.027593	0.00021	0.001934	0
17	3375.745	541021.9507	2.41E-03	0.023066	0.000469	0.000918	0.000035	0	0.000023
18	3645.102	2075705.707	7.91E-03	0.01024	0.000074	0.000004	0.000008	0.000007	0.000006
19	3934.726	8218154.449	2.69E-02	0.15868	0.00085	0.042389	0.000228	0.000932	0.00003
20	4259.572	292922.9968	8.18E-04	0.002384	0.000006	0.000166	0.000027	0.000027	0
21	4536.207	746133.2784	1.84E-03	0	0.000016	0.000852	0.000086	0.000022	0.000001
22	5070.419	2118456.192	4.17E-03	0.011117	0.000028	0.000017	0.000127	0	0.000003
23	5526.398	2152212.752	3.57E-03	0.020648	0.000002	0.002589	0.000007	0.000246	0
24	6124.603	1531931.186	2.07E-03	0.000179	0.000101	0.000729	0.000004	0.000038	0.000007

Table 2. Base bar results

For the case of the extended bar, 26 modal shapes were obtained, frequencies, the values of its kinetic energy, modal mass and the ratios between effective mass and total mass are located from column 5 to 10 and shows the corresponding bending and rotations in every axis. See table 2

Nr.	Freq [Hz]	Kin. energy [J]	Mod. mass[kg]	x_dir	y_dir	z_dir	rx_dir	ry_dir	rz_dir
1	7.168	62.827	6.20E-02	0.0000	0.4470	0.0000	0.0084	0.0000	0.9289
2	45.46	2760.015	6.77E-02	0.0000	0.1437	0.0000	0.0145	0.0000	0.0470
3	139.506	24249.611	6.31E-02	0.0026	0.0000	0.4462	0.0071	0.9144	0.0000
4	185.629	3983.962	5.86E-03	0.0001	0.0001	0.0025	0.0990	0.0063	0.0000
5	325.125	16265.5443	7.80E-03	0.0001	0.0904	0.0000	0.2711	0.0000	0.0094
6	510.936	27811.6236	5.40E-03	0.0001	0.1350	0.0007	0.3464	0.0001	0.0104
7	639.032	32437.5817	4.02E-03	0.0011	0.0180	0.0002	0.0858	0.0003	0.0010
8	659.904	51856.5119	6.03E-03	0.0005	0.0019	0.0068	0.0000	0.0021	0.0002
9	836.351	1107490.342	8.02E-02	0.0206	0.0001	0.1408	0.0011	0.0481	0.0000
10	1164.746	138064.9372	5.16E-03	0.0055	0.0002	0.0004	0.0017	0.0001	0.0000
11	1612.259	482088.9775	9.40E-03	0.1129	0.0001	0.0045	0.0008	0.0000	0.0000
12	1736.913	331548.343	5.57E-03	0.2108	0.0002	0.0000	0.0009	0.0010	0.0000
13	1827.061	168103.0733	2.55E-03	0.0158	0.0002	0.0009	0.0001	0.0000	0.0000
14	1990.571	207061.0781	2.65E-03	0.0049	0.0014	0.0001	0.0005	0.0000	0.0001
15	2470.776	575906.1364	4.78E-03	0.0795	0.0002	0.0000	0.0000	0.0007	0.0000
16	2614.323	1894243.612	1.40E-02	0.1664	0.0003	0.0332	0.0000	0.0085	0.0000
17	3046.7	2784730.954	1.52E-02	0.0123	0.0000	0.0944	0.0001	0.0099	0.0000
18	3306.275	559626.0927	2.59E-03	0.0150	0.0000	0.0088	0.0000	0.0003	0.0000
19	3424.118	788446.5403	3.41E-03	0.0028	0.0001	0.0180	0.0000	0.0020	0.0000
20	3523.959	11093407.88	4.53E-02	0.1258	0.0012	0.0698	0.0011	0.0008	0.0001
21	3958.933	586451.2396	1.90E-03	0.0054	0.0003	0.0001	0.0000	0.0001	0.0000
22	4327.172	1227090.502	3.32E-03	0.0004	0.0002	0.0024	0.0000	0.0003	0.0000
23	4629.095	2136219.114	5.05E-03	0.0010	0.0000	0.0001	0.0000	0.0000	0.0000
24	5322.793	1693449.03	3.03E-03	0.0004	0.0001	0.0007	0.0000	0.0001	0.0000
25	5434.293	2545555.035	4.37E-03	0.0009	0.0001	0.0002	0.0000	0.0000	0.0000
26	6072.409	1468834.386	2.02E-03	0.0004	0.0002	0.0008	0.0000	0.0002	0.0000

Table 3. Extended bar results

A comparison is made between the two tables of results, and the aim is try to find those modal forms that have similar frequencies. This comparison arises with the data of each one of the bars and try to observe the behaviour of each one of them and the difference of frequencies according to the modal test.

The values that are highlighted represent the coincidence in terms of frequency of each modal form, the values of blurred colour are those modal forms that do not coincide for any type of bar and have movements and / or torsions in other axes that are not important for our analysis. The kinetic energy that corresponds to the base bar, is approximately half that of the extended bar, remember that it has twice the width.

The results are taken from Tab. 2 and 3. The following Tab. 4 contains the simplified data of both bars specimens.

Base bar model A				Extended bar model B			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]	Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	5.221	31.3287	5.82E-02	1	7.168	62.827	6.20E-02
3	51.524	3064.9435	5.85E-02	2	45.46	2760.015	6.77E-02
4	237.365	3998.1207	3.60E-03	3	139.506	24249.611	6.31E-02
5	328.666	88405.2316	4.15E-02	4	185.629	3983.962	5.86E-03
6	345.524	18726.7622	7.95E-03	5	325.125	16265.5443	7.80E-03
7	568.017	31487.7302	4.94E-03	6	510.936	27811.6236	5.40E-03
8	734.091	19453.5681	1.83E-03	7	639.032	32437.5817	4.02E-03
9	885.176	47472.1108	3.07E-03	9	836.351	1107490.342	8.02E-02
10	1419.215	101697.62	2.56E-03	10	1164.746	138064.9372	5.16E-03
11	1485.519	1384242.156	3.18E-02	11	1612.259	482088.9775	9.40E-03
12	1941.648	235643.6121	3.17E-03	14	1990.571	207061.0781	2.65E-03
13	2119.367	147900.1424	1.67E-03	15	2470.776	575906.1364	4.78E-03
14	2235.712	303541.1776	3.08E-03	13	1827.061	168103.0733	2.55E-03
15	2565.035	989756.2294	7.62E-03	16	2614.323	1894243.612	1.40E-02
16	3134.661	376847.0712	1.94E-03	17	3046.7	2784730.954	1.52E-02
17	3375.745	541021.9507	2.41E-03	18	3306.275	559626.0927	2.59E-03
2	34.015	1382.5114	6.05E-02	19	3424.118	788446.5403	3.41E-03
18	3645.102	2075705.707	7.91E-03	20	3523.959	11093407.88	4.53E-02
20	4259.572	292922.9968	8.18E-04	22	4327.172	1227090.502	3.32E-03
21	4536.207	746133.2784	1.84E-03	23	4629.095	2136219.114	5.05E-03
22	5070.419	2118456.192	4.17E-03	24	5322.793	1693449.03	3.03E-03
23	5526.398	2152212.752	3.57E-03	25	5434.293	2545555.035	4.37E-03
24	6124.603	1531931.186	2.07E-03	26	6072.409	1468834.386	2.02E-03
19	3934.726	8218154.449	2.69E-02	21	3958.933	586451.2396	1.90E-03

Table 4. Frequencies comparison

The selection of the appropriate bar model to meet the requirements of BONATRANS is selected based on the proportion of modal mass and physical body mass. It means that this proportion that is greater represent the bar model with the highest energy displacement during the excitation simulation.

For both models A and B, the ratio corresponds to the total modal mass by the physical body mass of the bar

If the data that has been matched are used, as in Table 4, the difference in the ratio is not so great and can be considered as a very similar ratio as it is for other comparison cases. That is why is also included the sum of the modal mass, those modal forms that were not considered during the construction of table 4. The following table shows the ratio comparison between the modal A and B and indicates that ratio from Model B is the suitable one that must be considered to future constructions of the damping rubber.

Model A				Model B		
Modal mass[kg]	Body mass [kg]	Ratio		Modal mass[kg]	Body mass [kg]	Ratio
0.342	0.182	1.877		0.432	0.214	2.015

Table 5. Comparison the models by mass ratio

5.4 Modes shapes comparison

After ordering and comparing the modal forms that are similar in terms of frequency is good to have a visual check of the deformations that both bars are subjected. The following pictures show the mode shape and its respective frequencies.

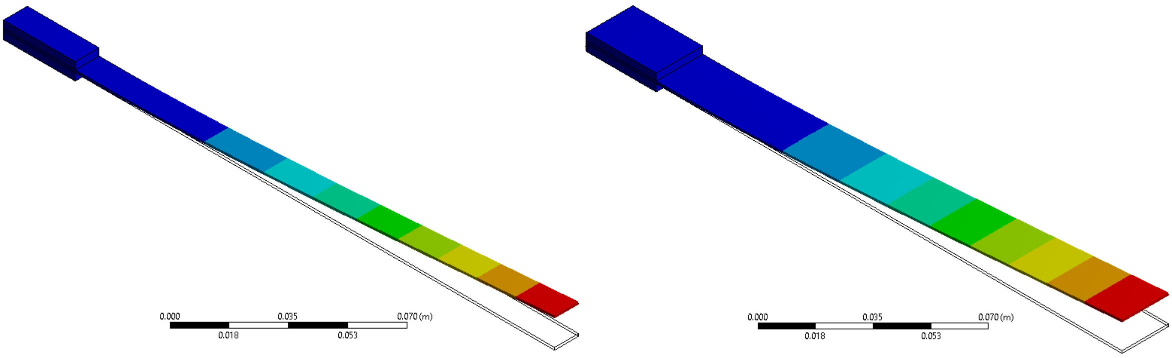


Figure 17 –Mode shape number 1 with 20.398 Hz and 20.467 Hz respectively

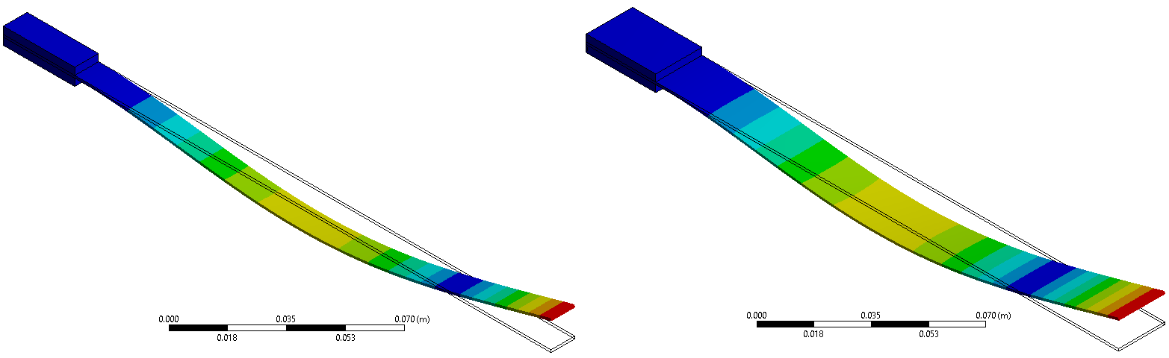


Figure 18 –Mode shape number 2 with 127.818 Hz and 128.223 Hz respectively

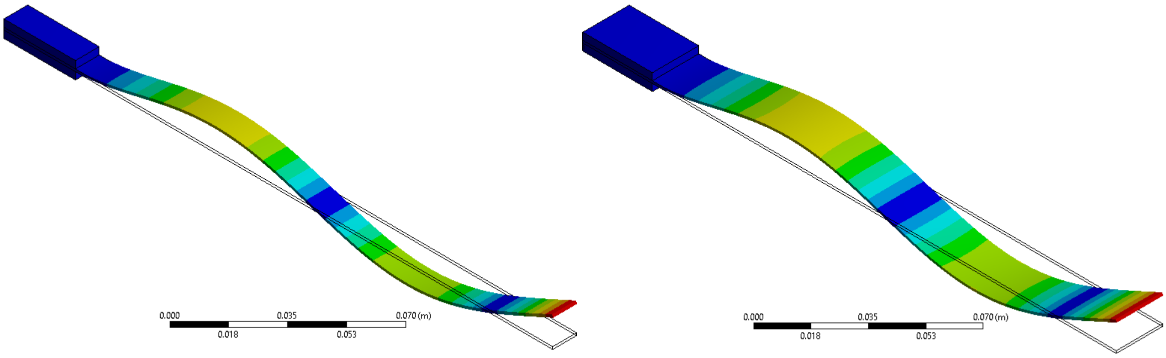


Figure 19 –Mode shape number 3 with 357.948 and 359.265 Hz respectively

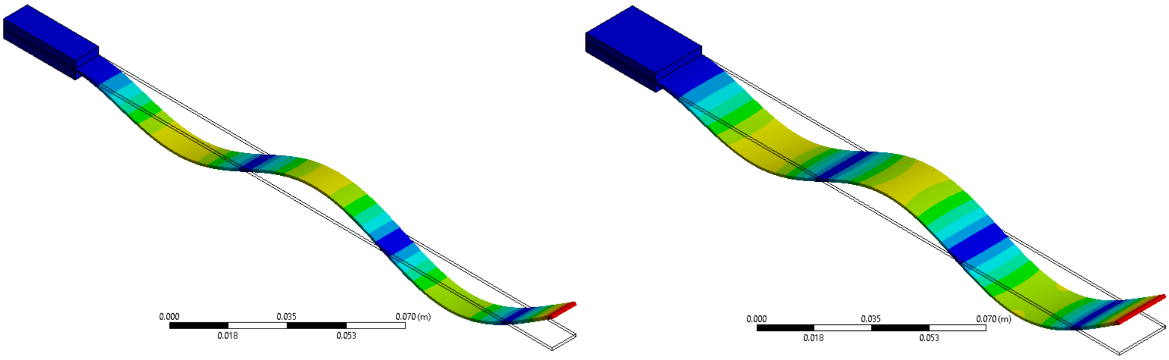


Figure 20 –Mode shape number 4 with 701.69 Hz and 704.995 respectively

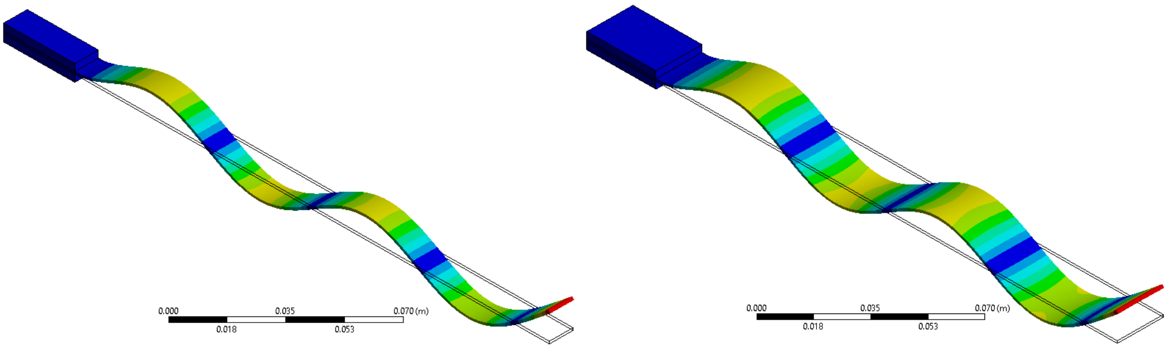


Figure 21 –Mode shape number 5 with 1160.588 Hz and 1167.559 Hz respectively

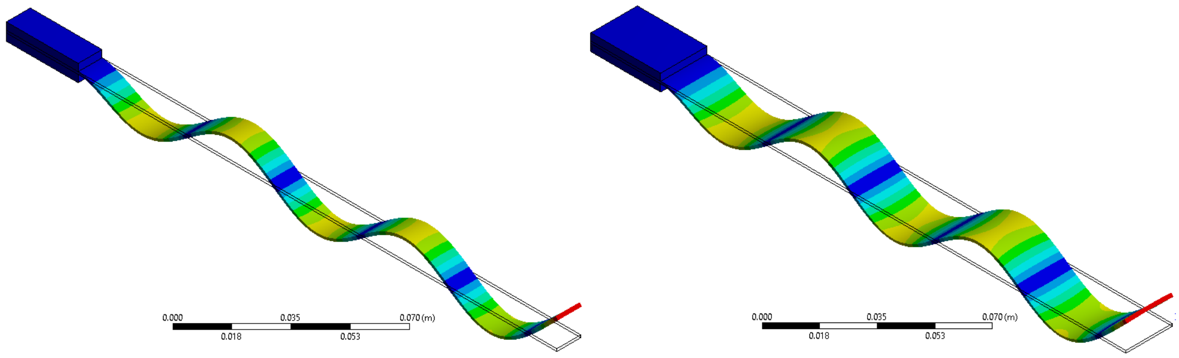


Figure 22 –Mode shape number 6 with 1734.959 Hz 1747.731 Hz respectively

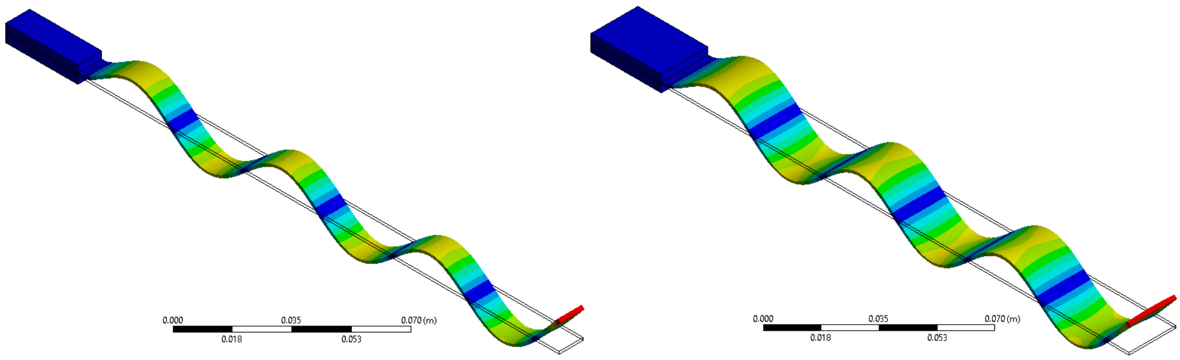


Figure 23 –Mode shape number 7 with 2425.257 Hz and 2446.167 Hz respectively

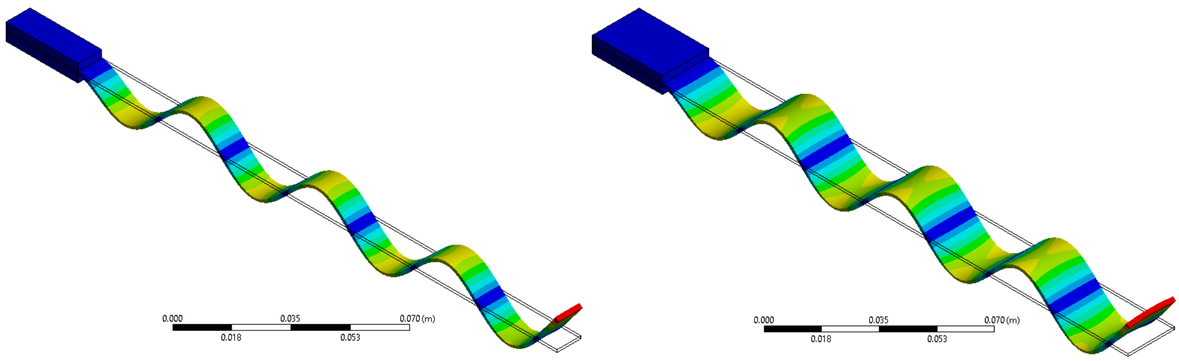


Figure 24 –Mode shape number 8 with 3231.936 Hz and 3263.262 Hz respectively

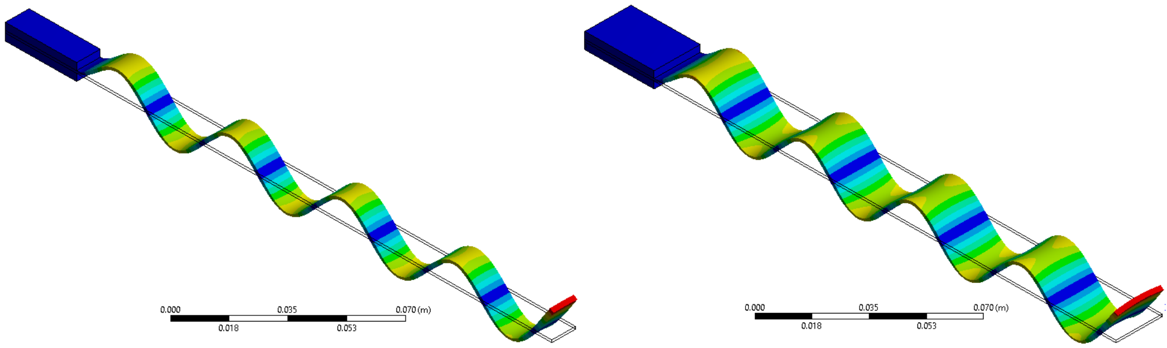


Figure 25 –Mode shape number 9 with 4155.426 Hz and 4199.188 Hz respectively

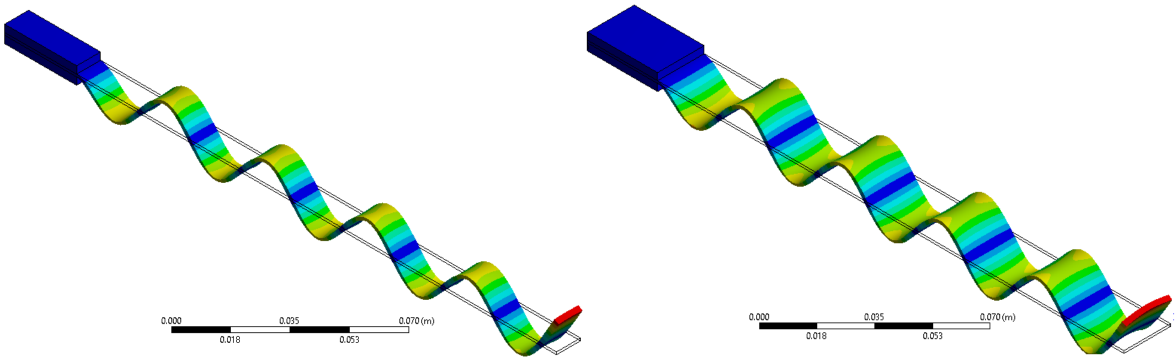


Figure 26 –Mode shape number 10 with 5196.107 Hz and 5253.931 Hz respectively

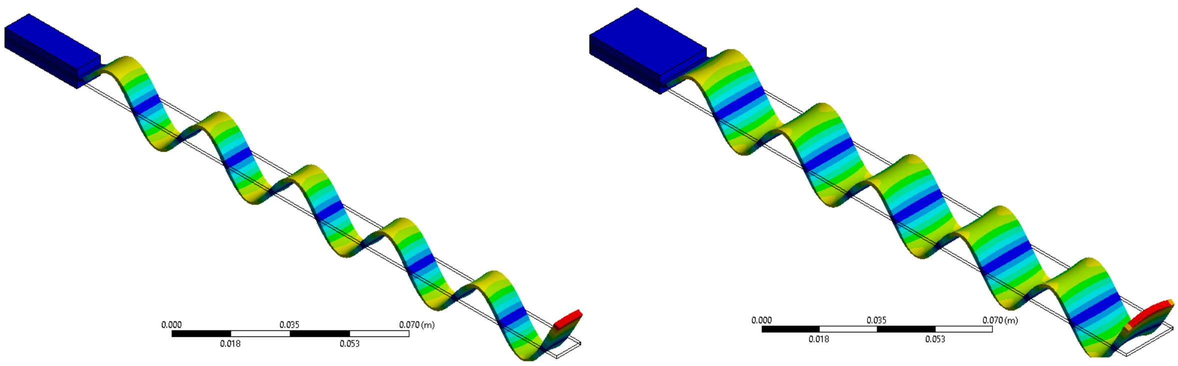


Figure 27 –Mode shape number 11 with 6354.297 Hz and 6427.316 Hz respectively

As part of the results of modal analysis of the variants of the bars and their mass distributions, the ratios are obtained due to the relation between the modal mass and the physical mass of each bar. n the following table, the total ratios of each bar model are shown specifically without mass and 4th mass type

Comparison of the ratios			
Bar model	A	B	C
No mass	2.28	2.42	
	2.28		2.01
		2.42	2.01
Mass 4th type	1.87	2.02	
	1.87		2.35
		2.02	2.35

Table 6. Ratios comparison

6 Proposed Specimen – Dynamic Verification

According to the result comparison, the suitable model is B with 4th mass distribution, so in order to show the comparison in more detail and make a deeper analysis, of the harmonic type. The selected bar type B shows the comparison in more detail and make a deeper analysis, of the harmonic type.

With an acceleration of 200 m / s is applied and simulate the vibration of the shaker. It should be remembered that is done the test for a pair of bars at the same time, one opposite the other, as explained above.

In this analysis is possible to see acceleration, amplitude and frequency response in the interval from 0 to 6500 Hz. The following graph 1, 2 and 3 shows the plot of the frequency response behavior.

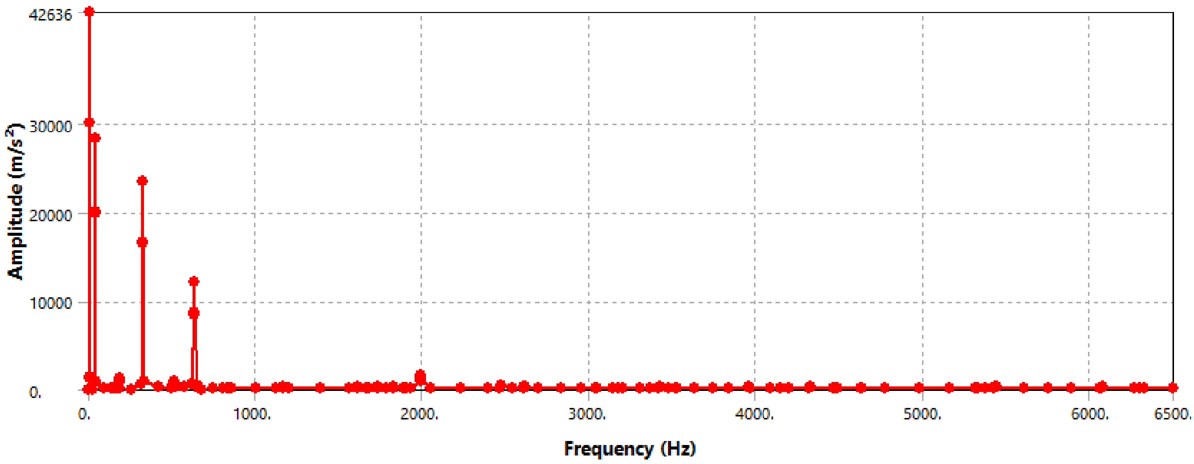


Figure 28 –FRF for acceleration (Model B)

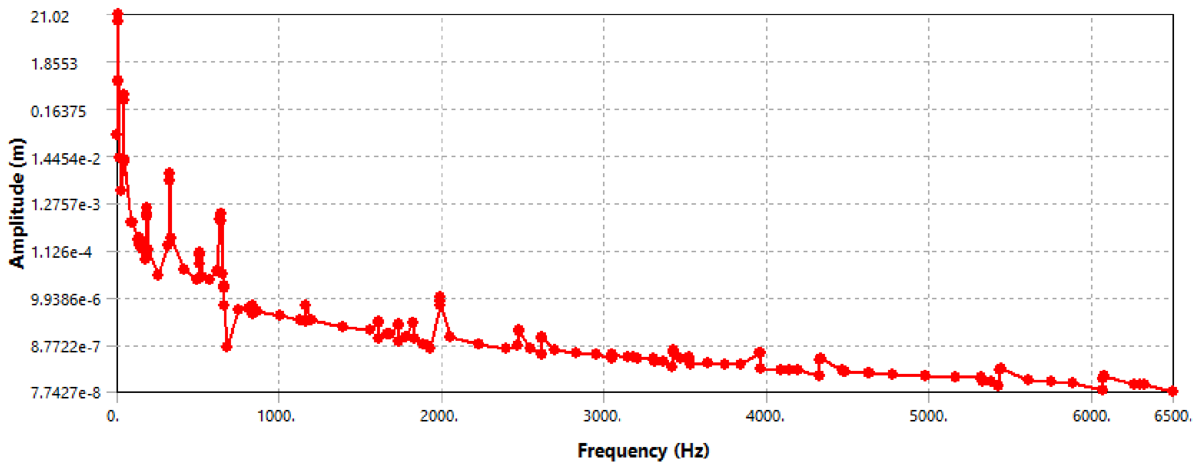


Figure 29 –FRF for displacement (Model B)

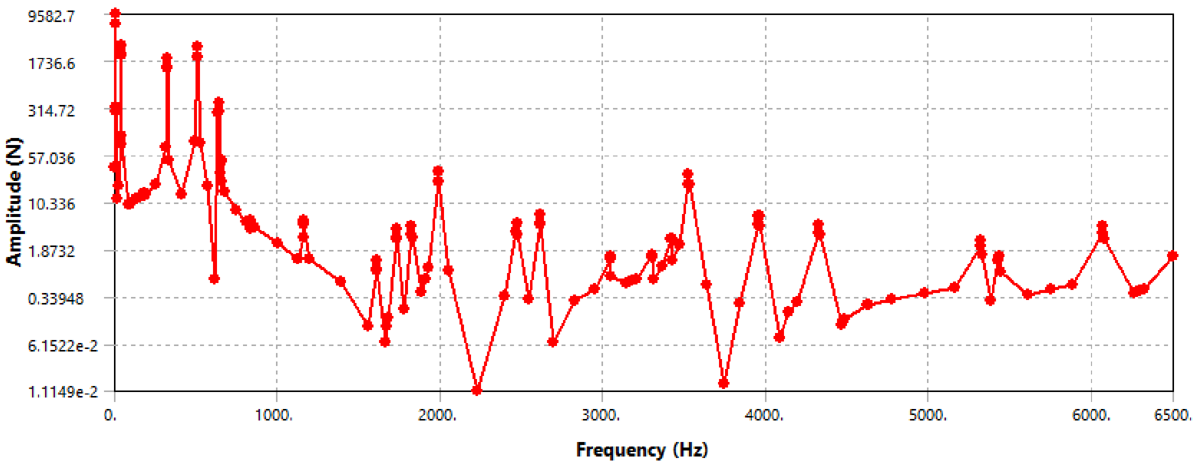


Figure 30 –FRF for force (Model B)

To complement the FRF, an analysis of the modal forms of the force has been included, which is given in the same range from 0 to 6500 Hz, and they are similar to the data thrown by the modal analysis. The following Tab.7 shows the notation of the correspondence mode shapes obtained from the FRF. The Fig. 31 shows the mode shapes form the FRF for force, which means that the peaks shown in figure 30 are those frequencies

Mode shape	Frequency [Hz]	Mode shape	Frequency [Hz]
1	7.168	10	1990.571
2	45.46	11	2470.776
3	325.125	12	2614.323
4	510.936	13	3523.959
5	639.032	14	3958.933
6	1164.746	15	4327.172
7	1612.259	16	5322.793
8	1736.913	17	5434.293
9	1827.061	18	6072.409

Table 7. FRM mode shapes notation

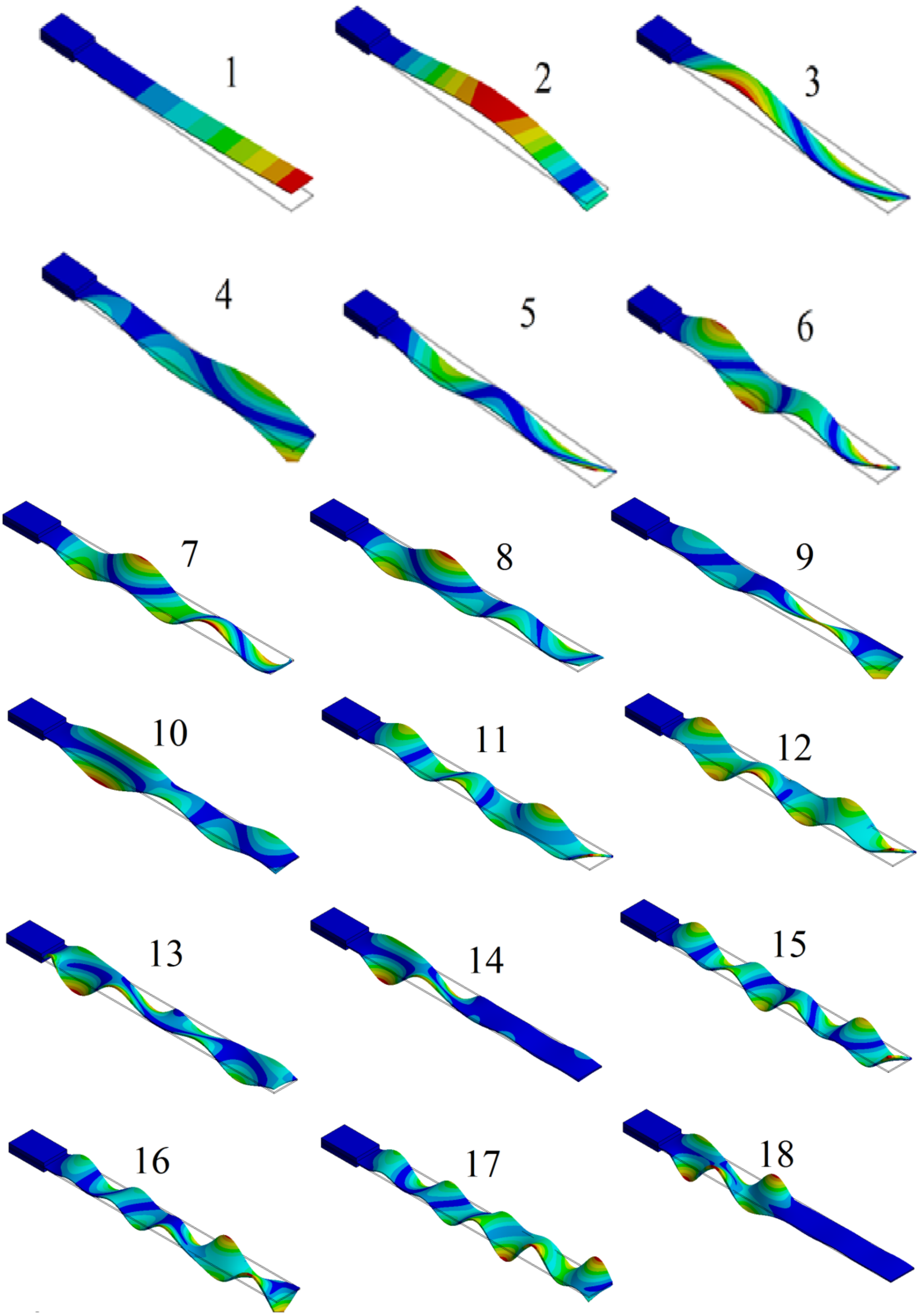


Figure 31 –FRF Mode shapes

7 CONCLUSION

The aim of this thesis is to support the process of computational development of a wheel damper. The goal of the work is to determine the suitable proposal of the metal piece of measurement specimen for dynamic damping properties determination. The dynamic testing and evaluation methodology is based on the standard ASTM E756-05: 2010 Standard Test Method for Measuring Vibration-Damping Properties of Materials which so far has been utilized in those kind of measurement. The methodology of the standard describes the determination of the modulus of elasticity in tension and shear based on the comparison of the intrinsic frequencies and loss factors of the basic non-damped fixed beams and beams with the applied damping layer.

At the very beginning it was necessary to perform the modal analysis of the initial base specimen, provided by the company, in order to find its natural frequencies and see its behaviour in the frequency range from 0 to 6500 Hz. That initial step must be done to obtain a reference level of crucial dynamic properties of currently utilized testing specimen.

The process of my work consists of three steps. The first is to propose a new dimensions of core metal piece, which will give the base for further variational analysis. The second step is to perform the modal analysis of all dimension and mass distribution of proposed variants. The third step is to check the dynamic properties and connected dynamic characteristics of the promising candidate among designed variants.

Firstly I worked with the base specimen, to see how it would behave, later two more models were proposed, varying their dimensions. Also distributed masses were included along the bars, and simulations were made. In addition to comparing the results of the modal analysis. Of all the simulations made following of the norm and the conditions for simulating previously established, I found the most suitable specimen so that in a future it will be able to reproduce in a real way, the vibration in the laboratory. Model B is the most suitable because it would be able to withstand the vibrations to which it would be subjected, and thus be able to determine the proper rubber.

In addition, 2 bar models with variants were proposed in their dimensions and included masses distributed throughout the structure. Giving place to a total of 12 different combinations of those that were selected that presented greater dissipation of energy at the moment of the simulation of said acceleration when it is included in the shaker.

The criterion that was taken to select the appropriate type of bar was based on the modal mass ratio, on body mass. In other words, the greater the deformation energy it may have, in order to be able to carry the material layer itself, it is to carry a layer of damping material, mainly rubber or compound, which the damping properties will have determined by the aforementioned measure.

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List of Appendices

APPENDIX A

This contents the comparison tables for all the model bars and type mass distributions.

Base bar type A				Extended bar type B			
Number	Freq. [Hz]	Kin. energy[J]	Mod. mass[kg]	Nr.	Freq [Hz]	Kin. energy [J]	Mod. mass[kg]
1	20.398	32.2272	3.92E-03	1	20.474	64.8279	7.84E-03
2	127.818	1265.6046	3.92E-03	2	128.262	2544.7066	7.84E-03
3	200.918	3154.6505	3.96E-03	4	394.516	15699.5927	5.11E-03
4	357.948	9924.0005	3.92E-03	3	359.324	19941.9362	7.83E-03
6	770.931	30653.902	2.61E-03	5	398.961	25065.9062	7.98E-03
5	701.69	38118.35	3.92E-03	6	704.955	76476.4471	7.80E-03
7	1160.588	104189.4658	3.92E-03	7	1167.151	208358.8576	7.75E-03
8	1244.929	123000.1436	4.02E-03	8	1189.9	139609.5948	5.00E-03
9	1734.959	232526.2372	3.91E-03	9	1746.5	462542.5974	7.68E-03
10	2315.959	274997.8955	2.60E-03	10	2004.131	379496.3868	4.79E-03
17	5440.687	1474371.347	2.52E-03	11	2394.941	956631.6222	8.45E-03
11	2425.257	453537.3657	3.91E-03	12	2443.428	895249.9974	7.60E-03
14	3870.466	759090.281	2.57E-03	13	2849.115	723927.7431	4.52E-03
12	3231.936	803505.6167	3.90E-03	14	3258.069	1569542.039	7.49E-03
13	3425.727	954185.4031	4.12E-03	15	3735.891	1164415.319	4.23E-03
15	4155.426	1324374.074	3.89E-03	16	4190.291	2551451.793	7.36E-03
16	5196.107	2063450.681	3.87E-03	18	5239.739	3903266.178	7.20E-03
18	6268.63	6127776.391	7.90E-03	20	6268.78	12251097.24	1.58E-02
19	6354.297	3073025.501	3.86E-03	21	6314.838	7188247.242	9.13E-03

Base bar type A				Extended bar type C			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]	Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
7	1160.588	104189.4658	3.92E-03	5	1140.62	403249.3035	1.57E-02
1	20.398	32.2272	3.92E-03	1	61.004	1732.5598	2.36E-02
2	127.818	1265.6046	3.92E-03	2	381.995	68004.3832	2.36E-02
3	200.918	3154.6505	3.96E-03	3	391.436	73073.8825	2.42E-02
4	357.948	9924.0005	3.92E-03	4	1069.236	532863.9573	2.36E-02
9	1734.959	232526.2372	3.91E-03	6	2094.996	2043195.971	2.36E-02
10	2315.959	274997.8955	2.60E-03	7	2352.619	2791217.285	2.56E-02
13	3425.727	954185.4031	4.12E-03	8	3438.659	3581352.7	1.53E-02
14	3870.466	759090.281	2.57E-03	9	3462.726	5565430.309	2.35E-02
16	5196.107	2063450.681	3.87E-03	10	5171.041	12350358	2.34E-02
15	4155.426	1324374.074	3.89E-03	11	5786.466	9710051.334	1.47E-02
17	5440.687	1474371.347	2.52E-03	12	6201.437	36358279.21	4.79E-02
18	6268.63	6127776.391	7.90E-03	13	6214.712	21007698.45	2.76E-02

Extended bar type B			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	20.474	64.8279	7.84E-03
2	128.262	2544.7066	7.84E-03
4	394.516	15699.5927	5.11E-03
5	398.961	25065.9062	7.98E-03
7	1167.151	208358.8576	7.75E-03
10	2004.131	379496.3868	4.79E-03
11	2394.941	956631.6222	8.45E-03
16	4190.291	2551451.793	7.36E-03
17	4674.471	1701016.393	3.94E-03
18	5239.739	3903266.178	7.20E-03
19	5673.795	2345639.815	3.69E-03
15	3735.891	1164415.319	4.23E-03
20	6268.78	12251097.24	1.58E-02

Extended bar type C			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	61.004	1732.5598	2.36E-02
2	381.995	68004.3832	2.36E-02
3	391.436	73073.8825	2.42E-02
4	1069.236	532863.9573	2.36E-02
5	1140.62	403249.3035	1.57E-02
6	2094.996	2043195.971	2.36E-02
7	2352.619	2791217.285	2.56E-02
8	3438.659	3581352.7	1.53E-02
9	3462.726	5565430.309	2.35E-02
10	5171.041	12350358	2.34E-02
11	5786.466	9710051.334	1.47E-02
12	6201.437	36358279.21	4.79E-02
13	6214.712	21007698.45	2.76E-02

Base bar type A			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	5.221	31.3287	5.82E-02
3	51.524	3064.9435	5.85E-02
4	237.365	3998.1207	3.60E-03
5	328.666	88405.2316	4.15E-02
6	345.524	18726.7622	7.95E-03
7	568.017	31487.7302	4.94E-03
8	734.091	19453.5681	1.83E-03
9	885.176	47472.1108	3.07E-03
10	1419.215	101697.62	2.56E-03
11	1485.519	1384242.156	3.18E-02
12	1941.648	235643.6121	3.17E-03
13	2119.367	147900.1424	1.67E-03
14	2235.712	303541.1776	3.08E-03
15	2565.035	989756.2294	7.62E-03
16	3134.661	376847.0712	1.94E-03
17	3375.745	541021.9507	2.41E-03
2	34.015	1382.5114	6.05E-02
18	3645.102	2075705.707	7.91E-03
20	4259.572	292922.9968	8.18E-04
21	4536.207	746133.2784	1.84E-03
22	5070.419	2118456.192	4.17E-03
23	5526.398	2152212.752	3.57E-03
24	6124.603	1531931.186	2.07E-03
19	3934.726	8218154.449	2.69E-02

Extended bar type B			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	7.168	62.827	6.20E-02
2	45.46	2760.015	6.77E-02
3	139.506	24249.611	6.31E-02
4	185.629	3983.962	5.86E-03
5	325.125	16265.5443	7.80E-03
6	510.936	27811.6236	5.40E-03
7	639.032	32437.5817	4.02E-03
9	836.351	1107490.342	8.02E-02
10	1164.746	138064.9372	5.16E-03
11	1612.259	482088.9775	9.40E-03
14	1990.571	207061.0781	2.65E-03
15	2470.776	575906.1364	4.78E-03
13	1827.061	168103.0733	2.55E-03
16	2614.323	1894243.612	1.40E-02
17	3046.7	2784730.954	1.52E-02
18	3306.275	559626.0927	2.59E-03
19	3424.118	788446.5403	3.41E-03
20	3523.959	11093407.88	4.53E-02
22	4327.172	1227090.502	3.32E-03
23	4629.095	2136219.114	5.05E-03
24	5322.793	1693449.03	3.03E-03
25	5434.293	2545555.035	4.37E-03
26	6072.409	1468834.386	2.02E-03
21	3958.933	586451.2396	1.90E-03

Base bar type A			
Nr.	Freq [Hz]	Kin. energy [J]	Mod. mass[kg]
2	34.015	1382.5114	6.05E-02
4	237.365	3998.1207	3.60E-03
5	328.666	88405.2316	4.15E-02
7	568.017	31487.7302	4.94E-03
8	734.091	19453.5681	1.83E-03
9	885.176	47472.1108	3.07E-03
10	1419.215	101697.62	2.56E-03
11	1485.519	1384242.156	3.18E-02
15	2565.035	989756.2294	7.62E-03
16	3134.661	376847.0712	1.94E-03
17	3375.745	541021.9507	2.41E-03
18	3645.102	2075705.707	7.91E-03
20	4259.572	292922.9968	8.18E-04
13	2119.367	147900.1424	1.67E-03
14	2235.712	303541.1776	3.08E-03
19	3934.726	8218154.449	2.69E-02
21	4536.207	746133.2784	1.84E-03
22	5070.419	2118456.192	4.17E-03

Extended bar type C			
Nr.	Freq [Hz]	Kin. energy [J]	Mod. mass[kg]
1	41.45	3377.40	9.96E-02
2	246.08	176817.60	1.48E-01
3	393.04	86737.25	2.84E-02
4	519.24	558295.41	1.05E-01
5	948.91	674807.58	3.80E-02
6	1250.65	351717.27	1.14E-02
7	1757.50	2134254.93	3.50E-02
8	2461.05	22614188.03	1.89E-01
9	2580.53	3082500.75	2.35E-02
10	2840.03	8292189.20	5.21E-02
11	3434.37	3535030.59	1.52E-02
12	3591.11	8754685.58	3.44E-02
13	3746.28	12464902.58	4.50E-02
16	5682.74	15692336.66	2.46E-02
17	5941.44	45117373.75	6.48E-02
18	6365.07	11583544.14	1.45E-02
14	4587.64	13675989.64	3.29E-02
15	4966.70	58196756.25	1.20E-01

Extended bar type B			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
2	45.46	2760.015	6.77E-02
3	139.506	24249.611	6.31E-02
4	185.629	3983.962	5.86E-03
6	510.936	27811.6236	5.40E-03
7	639.032	32437.5817	4.02E-03
8	659.904	51856.5119	6.03E-03
12	1736.913	331548.343	5.57E-03
15	2470.776	575906.1364	4.78E-03
16	2614.323	1894243.612	1.40E-02
17	3046.7	2784730.954	1.52E-02
18	3306.275	559626.0927	2.59E-03
1	7.168	62.827	6.20E-02
5	325.125	16265.5443	7.80E-03
9	836.351	1107490.342	8.02E-02
10	1164.746	138064.9372	5.16E-03
11	1612.259	482088.9775	9.40E-03
13	1827.061	168103.0733	2.55E-03
19	3424.118	788446.5403	3.41E-03

Extended bar type C			
Number	Freq [Hz]	Kin. energy[J]	Mod. mass[kg]
1	41.447	3377.3955	9.96E-02
2	246.084	176817.6002	1.48E-01
3	393.041	86737.2511	2.84E-02
4	519.243	558295.4149	1.05E-01
5	948.914	674807.5849	3.80E-02
6	1250.645	351717.2737	1.14E-02
7	1757.502	2134254.925	3.50E-02
8	2461.054	22614188.03	1.89E-01
9	2580.532	3082500.749	2.35E-02
10	2840.028	8292189.201	5.21E-02
12	3591.106	8754685.577	3.44E-02
13	3746.284	12464902.58	4.50E-02
14	4587.638	13675989.64	3.29E-02
15	4966.703	58196756.25	1.20E-01
16	5682.744	15692336.66	2.46E-02
17	5941.438	45117373.75	6.48E-02
18	6365.067	11583544.14	1.45E-02
11	3434.373	3535030.591	1.52E-02

APPENDIX B

This contents the force FRF for the most promising model bars and type mass distributions.